1. Binary integers.

(a) (4 points) Express the decimal number 432 in hexadecimal.

\[ 0x1b0 \]

(b) (4 points) Interpret the hexadecimal bit pattern \( 0xffcc \) as a 16-bit 2’s complement number. What is its decimal value?

\[ -52 \]

(c) (4 points) Suppose that \( a \) is a \( k \)-bit 2’s complement number and \( b \) is its \( k \)-bit additive inverse (negative). If we reinterpret their bit patterns as \( k \)-bit unsigned numbers, add them, and keep any carry out of the leftmost place, what number do we get?

If \( a \) is zero we get zero. Otherwise we get \( 2^k \). That’s why it’s called 2’s complement.

(d) (4 points) Suppose we use an integer divide instruction to divide \(-20\) by \(-3\). What will be the quotient? What will be the remainder?

Quotient 6; remainder \(-2\).

2. Floating point.

(a) (5 points) What are subnormal (denormal) numbers? What purpose do they serve?
They're numbers close to zero that have an implied zero bit to the left of the binary point instead of an implied one, and therefore a reduced number of significant bits. They provide gradual underflow, so we don't have a (comparatively) big gap between zero and the smallest magnitude numbers.

(b) (5 points) A positive single precision floating point number with significand $f$ and exponent $e$ has value $1.f \times 2^{e-127}$. The value 127 is called the bias. What purpose does it serve?

It ensures that larger numbers always have “larger” exponent values, when interpreted as an unsigned integer. This allows us to use similar comparator circuits for integer and FP numbers.

(c) (5 points) If $x$, $y$, and $z$ are C variables of type double, will $(x + y) + z$ have the same value as $x + (y + z)$? Explain.

Not necessarily. If, for example, $x$ and $y$ have opposite sign and approximately the same large magnitude, but $z$ has much smaller magnitude, then $z$ may be lost when added to $y$ in the second expression, but may be quite significant in the first, if $x$ and $y$ roughly cancel out.

3. Memory layout.

(a) (10 points) What does the following program print on a 32-bit x86 machine? Explain. (Be careful! This problem tests your understanding of array layout, alignment, and endian-ness (byte order). Remember that fields of a union lie on top of each other.)

```c
#include <stdio.h>

int main() {
    int i;
    union {
        unsigned char A[8];
        struct {
            unsigned char c;
            int n;
        } S;
    } U;

    for (i = 0; i < 8; i++) U.A[i] = 0;
    U.A[5] = 0x3;
    printf("%x\n", U.S.n);  /* print U.S.n in hex */
}
```

It prints 300. The 8 bytes of $A$ lie on top of $S$. There is a 3-byte hole after $c$, so that $n$ can be longword-aligned. $A[5]$ therefore coincides with the second byte of $n$ in memory. Because the x86 is a little-endian machine, this is the second-to-least significant byte. The 00 part of the answer is the LSB; the 3 is the second-to-least.
(b) (5 points) Here is the header for a C function to copy row $i$ of a contiguous-layout $N \times N$ array $M$ into an $N$-element array $V$. Fill in the body of the code.

```c
void copy_row(int *M, int i, int N, int *V) {
    int j; for (j = 0; j < N; j++) V[j] = M[N*i+j];
    Equivalently:
    int *end = V+N; for (M+=(N*i); V < end; M++, V++) *V = *M;
}
```

(c) (4 points) Explain the difference between the C declarations `int *a[n]` and `int (*a)[n]`.

- `int *a[n]` is an array of $n$ pointers to integers; `int (*a)[n]` is a pointer to an array of $n$ integers.

(d) (4 points) What is the value of $n$ after executing the following C code on a 32-bit x86? Explain.

```c
int A[100];
int *p = &A[10];
int *q = &A[20];
int n = q - p;
```

10. The size of the array elements is immaterial. C computes the numeric difference in indices.

4. Understanding assembler.

(a) (4 points) Why do C compilers usually arrange function arguments as if they had been pushed in reverse order (last one first)?

So the first argument will be at a known offset from the frame pointer. This is necessary for functions that accept variable numbers of arguments.

(b) (4 points) Why do `switch` statements exist? Why not just use `if...else if...else`?

Because compilers can generate particularly efficient code for `switch` statements using techniques like array indexing, hashing, and binary search.

(c) (4 points) Name four different instruction set architectures currently in widespread use.

I’ve mentioned the following in class: ARM, x86, IA-64 (Itanium), Sparc, MIPS, Power/PowerPC, and IBM z.

(d) (5 points) Some ISAs use the term `branch` for an instruction that changes the program counter by adding or subtracting an offset, and `jump` for an instruction that changes the program counter by setting it to some absolute value. Why might it be desirable to have both?

A branch instruction can be encoded in fewer bits. The linker can place it (together with its target) anywhere in memory without having to change the instruction encoding. At the same time, a branch can only reach a limited distance from the current location. A jump instruction requires more bits for its encoding, and must be modified when the linker chooses where to put the target location. On the plus
side, a jump can reach an arbitrary location. Commonly, compilers use branches within a compilation unit and jumps between compilation units.

(e) (5 points) On most systems, the subroutine call stack grows downward, toward smaller addresses. Is this merely a software convention on the x86, or is it reflected somehow in the hardware? Explain.

It’s reflected in the hardware: the push, call, and enter instructions subtract from %esp; the pop, leave, and ret instructions add to it.

(f) (Extra Credit; 6 points max) What does the following function do? (Note: There is a simple high-level answer to this question. Do not give me a detailed instruction-by-instruction description of the code. Be sure to pay careful attention to the difference between %eax and %edx.)

foo:

```
pushl %ebp
movl %esp, %ebp
movl 8(%ebp), %edx
popl %ebp
leal (%edx,%edx,4), %eax
leal (%edx,%eax,4), %eax
ret
```

It multiplies its argument by 21 and returns the result.

(g) (Extra Credit; 4 points max) Describe two problems posed by the ability to execute code in the stack on an x86 machine.

1. It leaves programs with inadequate bounds checking vulnerable to buffer overflow attacks.
2. It makes it difficult for hardware designers to implement the first-level caches: we have to notice when a store instruction modifies a location currently in the instruction cache.

5. Processor implementation.

(a) (5 points) Explain the difference between combinational and sequential circuits.

Combinational circuits implement a pure Boolean function from inputs to outputs. They have no feedback (cycles in their wiring). Sequential circuits may have multiple stable states, allowing them to store values. They can be implemented with feedback loops or other forms of memory.

(b) (4 points) What are the time complexity and space complexity of a carry-lookahead adder? (Use big-O notation.)

O(log n) time, O(n log n) space, where n is the number of bits in the numbers to be added.

(c) (5 points) Name, in order, the five stages of the Y86 PIPE implementation in the textbook.

Fetch, Decode, Execute, Memory, Writeback.
(d) (5 points) Why does the Y86 PIPE implementation predict its way through conditional branches, but not through `ret` instructions? More specifically, it speculatively fetches instructions from the target of a conditional branch, but stalls the pipeline at a `ret`. Why?

A conditional branch instruction contains the target address of the branch, so it's available in the first stage of the pipeline. A `ret` instruction obtains the return address from the stack, so it's not available until the fourth stage of the pipeline.

(e) (5 points) Briefly explain the purpose of the reorder buffer (ROB) in an out-of-order processor. (The book calls this the retirement unit.)

It serves to make exceptions precise. An instruction is not allowed to commit (retire) until all instructions from earlier in program order have committed. Exceptions, if any, are generated at commit time.

(f) (Extra Credit; 4 points max) The PowerPC has multiple sets of condition codes. An instruction like `test` or `compare` specifies which set of codes to set. A branch instruction specifies which set of codes to inspect. What advantage might this provide over the single set of the x86?

Like software register renaming, multiple condition codes allow the compiler to avoid artificial dependences among instructions. The machine can have more than one set of test-and-branch instructions “in flight” at the same time.

(g) (Extra Credit; 6 points max) Ten years ago, at the peak of aggressive single-core processor design, the PowerPC G5 and the Pentium 4 had comparable computational power, comparable implementation complexity, comparable heat dissipation, and comparable manufacturing cost. The x86 has more instructions, but not a lot more. Yet the x86 is supposedly a CISC ISA and the PowerPC is supposedly a RISC ISA. Is this proof that the CISC v. RISC debate doesn’t matter any more?

Maybe. The G5 had uniform length instructions, a shorter pipeline, wider issue width (superscalar-ness), many more architectural registers, and 64-bit native operation. It kept many more instructions in flight at one time. It had no need for front-end translation from machine language to micro-ops. By doing more per cycle, it achieved comparable computational power with a substantially slower clock. These differences suggest that the PowerPC and its variants might enjoy somewhat lower development cost for new versions, lower vulnerability to mispredicted branches, and perhaps more opportunity for further improvements in instruction-level parallelism. Indeed, IBM continues to develop Power processors that outperform Intel’s top-of-the line x86 offerings—but not by much.