

OVERVIEW Here's a version of the infamous "Venn Diagram". Its rectangles share the "top edge" of the page...

LANGUAGE	GRAMMAR	MACHINE	CXTY
Finite	List	Enumer.	$O(n)$
Reg. Exp $(ab^*)^* (0^*1^*)^*$	$S \rightarrow aT$ $S \rightarrow b$	RE=DFA =NFA=RE	$O(n)$
Context Free	$S \rightarrow aSb$	NPDA > DPDA > Lin. DPDA	Poly: $O(n^3)$ $O(n)$
Context Sensitive	$aS \rightarrow bTSa$	LBA	Expon.
Recursively Enumerable	$aT \rightarrow b$	TM	Undeci- dable

DEFINITIONS TYPES 0-3 (CHOMSKY HIERARCHY)

Erasing Convention (EC). Limit erasing to single $S \rightarrow \lambda$ production, with S the start symbol. If that happens, S doesn't appear on RHS of any other productions.

Given that (have to convince self it's OK),

G is Context Sensitive (Type 1): obeys EC, and otherwise for all $\alpha \rightarrow \beta$, β is at least as long as α .

G is Context Free (Type 2): obeys EC, for all $\alpha \rightarrow \beta$, α is a single nonterminal.

G is Regular (Type 3): obeys EC and for all $\alpha \rightarrow \beta$, α is a single nonterminal and β is of form t or tW , t a terminal, W non-terminal.

REGULAR, CONTEXT FREE

$G = (V, T, S, P)$ with $V = \{0, 1, S\}, T = \{0, 1\}, P$

$S \rightarrow 0S \quad S \rightarrow 1$

$L = (ab)^*c \mid db^*$: $G = (V, T, S, P)$ with
 $V = \{a, b, c, d, S, B, TS\}, T = \{a, b, c, d\}, P$

$S \rightarrow d \quad S \rightarrow dT \quad T \rightarrow b \quad T \rightarrow bT$

$S \rightarrow c \quad S \rightarrow aQ \quad Q \rightarrow bX \quad X \rightarrow c \quad X \rightarrow aQ$

I think! This is Type 3 = RE, Scanners, we know this!

L is all non-empty strings with even number of 1's: $G = (V, T, S, P)$ with $V = \{1, S\}, T = \{1\}, P$

$S \rightarrow SS \quad S \rightarrow 11$

Not Regular (RHS has two NT or two T). Note other grammars but...

Type 1 Grammar

L is $\{a^n b^n c^n \mid n \geq 1\}$. $G = (V, T, S, P)$ with
 $V = \{a, b, c, S, B, C\}$, $T = \{a, b, c\}$

$S \rightarrow aSBC$ $S \rightarrow aBC$ $CB \rightarrow BC$ $aB \rightarrow ab$ $bB \rightarrow bb$ $bC \rightarrow bc$ $cC \rightarrow cc$
 S $aSBC$ $aaBCBC$ $aaBBCC$ $aabBCC$ $aabbCC$ $abbcC$ $aabbcc$

Not CFG (note multiple-symbol LHSs).

LBA recognizes since non-shrinking productions mean no intermediate sentential form is longer than original input string.

Type 0 Grammar (not CS)

L is empty string λ (or 1) with set of strings consisting of odd number of 0s, > 3 .

$G = (V, T, S, P)$ with

$V = \{0, A, B, E, F, W, X, Y, Z, S\}$, $T = \{0\}$, and P (note shrinking productions)

$S \rightarrow FA$ $S \rightarrow FBA$ $FB \rightarrow FOEBO$ $EB \rightarrow XBY$ $OX \rightarrow OX$
 $EB \rightarrow O$ $YO \rightarrow OY$ $FX \rightarrow FOW$ $YA \rightarrow ZOA$ $WO \rightarrow OW$
 $OZ \rightarrow ZO$ $WBZ \rightarrow EB$ $F \rightarrow 1$ $A \rightarrow 1$

Derivation: $S \Rightarrow FA \Rightarrow^* \lambda\lambda$ gives λ .

S FBA FOEBOA FOXBYOA * FXOBOYA * FOWOBOZOA
* FOOWBZOOA FOOEB00A F00000A * 00000

(Note X,Y,W,Z march across the 0 string, adding one on each side. Very TM- like.

TURING MACHINE

Finite alphabet, Finite number of non-blank cells, finite but unbounded memory. Cell blank or one symbol. FSA reads one cell at any given moment, then either halts or takes these actions:

1. Print a symbol from alphabet on the cell read (possibly same symbol).
2. Go to next state (could be same)
3. Move the Read-Write head one cell left or right.

Describe by set of quintuples $\{s, r, p, n, d\}$ for (current state, current symbol, symbol printed, next state, direction). In a deterministic TM no two quintuples have same s, r . In nondeterministic, there can be a choice of actions: this models parallelism or precognition but doesn't add power (new languages accepted).