

# PARAMODULATION - REASONING WITH EQUALITIES

" = " Useful not only for math, but ordinary reasoning ... "only", "no-one but", "everyone but", ...

"Kennedy was the only Catholic president"

$$P(K) \wedge C(K) \wedge \forall x. P(x) \wedge C(x) \Rightarrow x = K$$

"John respects everyone but himself"

$$(\forall x. \neg(x = J) \Rightarrow R(J, x)) \wedge \neg R(J, J)$$

We could use axioms for equality (without adding inference rules), but using paramodulation (substitution of equals) gives simpler, clearer proofs.

(See detailed notes for axioms, derived from the FOL schema

$$(\sigma = \tau) \wedge \varphi \Rightarrow \varphi_{\sigma/\tau}$$

which we apply to each promise clause  $\varphi$ )

## Simple example

$$\text{Wrote}(\text{Twain}, \text{Huck.Finn}) \vee \text{Wrote}(\text{Clemens}, \text{Huck.Finn})$$

P

$$\text{Twain} = \text{Clemens}$$

$$\text{Wrote}(\text{Clemens}, \text{Huck.Finn}) \vee \text{Wrote}(\text{Clemens}, \text{Huck.Finn})$$

redundant step if we use sets of literals for clauses

F

$$\text{Wrote}(\text{Clemens}, \text{Huck.Finn})$$

If we had

$$\text{Twain} = \text{Clemens} \vee \text{Twain} = \text{Shaw}$$

we would "carry along" the extra disjunct, with result

$$\text{Wrote}(\text{Clemens}, \text{Huck.Finn}) \vee \text{Twain} = \text{Shaw}$$

If there are variables, we can unify:

$$1. \neg \text{Wrote}(x, \text{Huck.Finn}) \vee \text{Twain} = x$$

$$2. \text{Father-of}(y, f(y))$$

P

$$p[1b_2, 2]: \text{Father-of}(y, \text{Twain}) \vee \neg \text{Wrote}(f(y), \text{Huck.Finn})$$

There are 4 other paramodulants!

In general, paramodulating

$$\sigma = \tau \vee \alpha_1 \vee \dots \vee \alpha_m$$

into

$$\beta_1(\sigma') \vee \beta_2 \vee \dots \vee \beta_n$$

Note: positive equality literal

$\sigma'$  occurs somewhere in  $\beta_1$ .

- unify  $\sigma$  with some term  $\sigma'$  in the second clause  
→ unifier  $\gamma$
- apply unifier  $\gamma$  throughout both clauses
- substitute  $\tau\gamma$  for  $\sigma'\gamma$
- infer (as "paramodulant") the clause consisting of all the literals of the two clauses (after application of unifier  $\gamma$  and substitution of  $\tau\gamma$  for  $\sigma'\gamma$ ), except for the equality literal

Symbolically:

$$\frac{\sigma = \tau \vee \alpha_1 \vee \dots \vee \alpha_m, \quad \beta_1(\sigma') \vee \beta_2 \vee \dots \vee \beta_n}{\alpha_1'' \vee \dots \vee \alpha_m'' \vee \beta_1''(\tau\gamma) \vee \beta_2'' \vee \dots \vee \beta_n''}$$

where double-primed symbols correspond to unprimed symbols after substitution  $\gamma$  (which is the m.g.u. of  $\sigma, \sigma'$ )

E.g. (p. 8 detailed notes) paramodulate  
 $F(x, B) = C \vee P(x, y)$  into  $Q(F(A, z)) \vee R(w, z) \vee S(F(A, B))$

For completeness, it turns out we still need

one axiom:

$$x = x$$

(Try proving an obvious truth like  $A = A$  without it!)

The princess and the pea (Earl Hunt, AI)

$M(x) \dots x$  is a man,  $\neg M(x) \dots x$  is a woman  
 $C(x) \dots x$  is a commoner

$K \dots$  the king,  $Q \dots$  the queen,  $P \dots$  the princess  
 $f(x, y) \dots$  the daughter of  $x$  and  $y$  (a function)

$A, B \dots$  some unidentified individuals

$D(x) \dots x$  can detect a pea under 20 featherbeds

Given

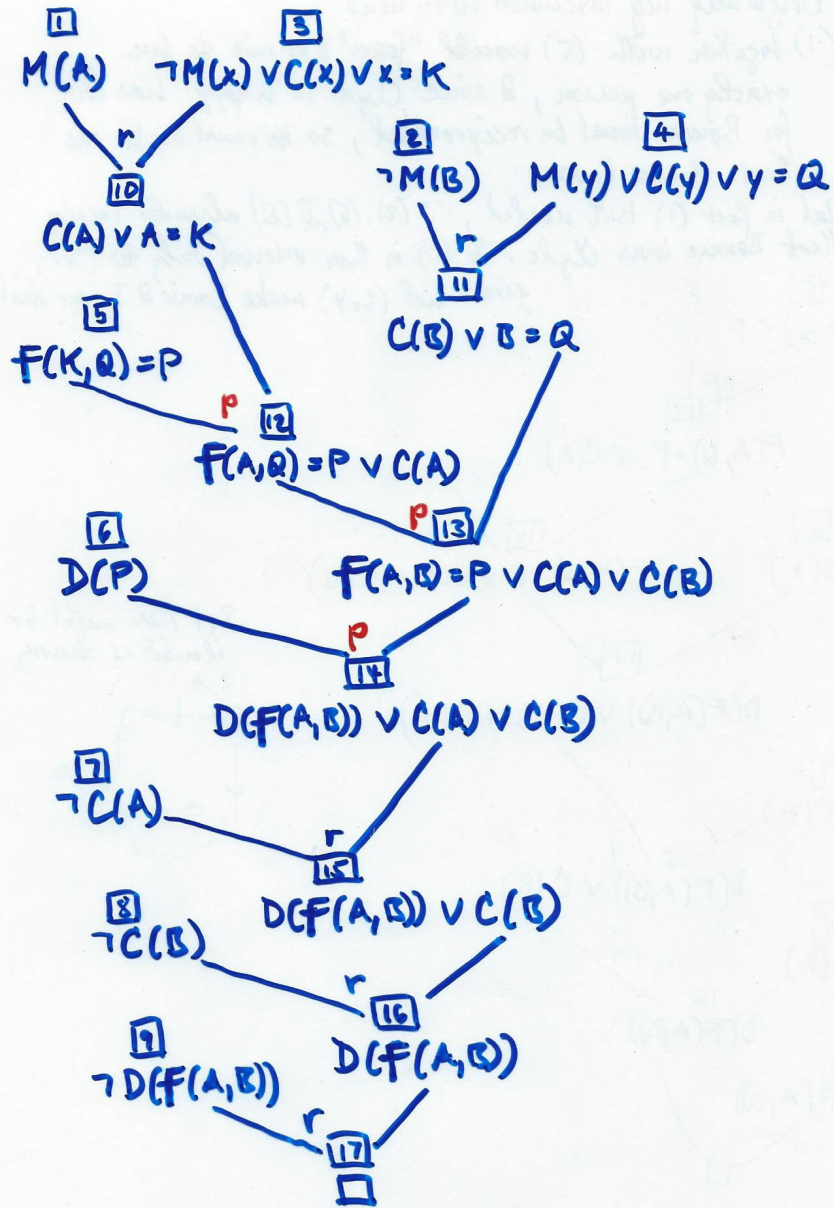
1.  $M(A)$
2.  $\neg M(B)$
3.  $\neg M(x) \vee C(x) \vee x = K$
4.  $M(y) \vee C(y) \vee y = Q$
5.  $f(K, Q) = P$
6.  $D(P)$

To be proved

$$(\neg C(A) \wedge \neg C(B)) \Rightarrow D(f(A, B))$$

Denial

7.  $\neg C(A)$
8.  $\neg C(B)$
9.  $\neg D(f(A, B))$



### Unrequited love : QA example

See printed notes for details

1. Everyone loves someone
2. Anyone whose love is not returned is unhappy
- 3, 4. AL loves Bonnie but not Diane
5. Bonnie loves at most one person (use identity)
- 6, 7. Clyde loves Bonnie & Diane
8. Clyde is happy

Who is unhappily in love with whom?

$\exists x \exists y. \neg H(x) \wedge L(x, y)$

Denial :  $\forall x \forall y. H(x) \vee \neg L(x, y)$

Use :  $H(x) \vee \neg L(x, y) \vee \text{Ans}(x, y)$

carry through proof

