

OVERVIEW Here's a version of the infamous "Venn Diagram". Its rectangles share the "top edge" of the page...

LANGUAGE	GRAMMAR	MACHINE	CXTY
Finite	List	Enumer.	$O(n)$
Reg. Exp $(ab^*)^*   (0^*1^*)^*$	$S \rightarrow aT$ $S \rightarrow b$	RE=DFA =NFA=RE	$O(n)$
Context Free	$S \rightarrow aSb$	NPDA > DPDA > Lin. DPDA	Poly: $O(n^3)$ $O(n)$
Context Sensitive	$aS \rightarrow bTSa$	LBA	Expon.
Recursively Enumerable	$aT \rightarrow b$	TM	Undeci- dable

## DEFINITIONS TYPES 0-3 (CHOMSKY HIERARCHY)

*Erasing Convention (EC)*. Limit erasing to single  $S \rightarrow \lambda$  production, with  $S$  the start symbol. If that happens,  $S$  doesn't appear on RHS of any other productions.

Given that (have to convince self it's OK),

G is Context Sensitive (Type 1): obeys EC, and otherwise for all  $\alpha \rightarrow \beta$ ,  $\beta$  is at least as long as  $\alpha$ .

G is Context Free (Type 2): obeys EC, for all  $\alpha \rightarrow \beta$ ,  $\alpha$  is a single nonterminal.

G is Regular (Type 3): obeys EC and for all  $\alpha \rightarrow \beta$ ,  $\alpha$  is a single nonterminal and  $\beta$  is of form  $t$  or  $tW$ ,  $t$  a terminal,  $W$  non-terminal.

## REGULAR, CONTEXT FREE

$G = (V, T, S, P)$  with  $V = \{0, 1, S\}, T = \{0, 1\}, P$

$S \rightarrow 0S \quad S \rightarrow 1$

$L = (ab)^*c \mid db^*$ :  $G = (V, T, S, P)$  with  
 $V = \{a, b, c, d, S, B, TS\}, T = \{a, b, c, d\}, P$

$S \rightarrow d \quad S \rightarrow dT \quad T \rightarrow b \quad T \rightarrow bT$

$S \rightarrow c \quad S \rightarrow aQ \quad Q \rightarrow bX \quad X \rightarrow c \quad X \rightarrow aQ$

I think! This is Type 3 = RE, Scanners, we know this!

$L$  is all non-empty strings with even number of 1's:  $G = (V, T, S, P)$  with  $V = \{1, S\}, T = \{1\}, P$

$S \rightarrow SS \quad S \rightarrow 11$

Not Regular (RHS has two NT or two T). Note other grammars but...

## Type 1 Grammar

$L$  is  $\{a^n b^n c^n \mid n \geq 1\}$ .  $G = (V, T, S, P)$  with  
 $V = \{a, b, c, S, B, C\}$ ,  $T = \{a, b, c\}$

$S \rightarrow aSBC$   $S \rightarrow aBC$   $CB \rightarrow BC$   $aB \rightarrow ab$   $bB \rightarrow bb$   $bC \rightarrow bc$   $cC \rightarrow cc$   
 $S$   $aSBC$   $aaBCBC$   $aaBBCC$   $aabBCC$   $aabbCC$   $abbcC$   $aabbcc$

Not CFG (note multiple-symbol LHSs).

LBA recognizes since non-shrinking productions mean no intermediate sentential form is longer than original input string.

## Type 0 Grammar (not CS)

L is empty string  $\lambda$  (or 1) with set of strings consisting of odd number of 0s,  $> 3$ .

$G = (V, T, S, P)$  with

$V = \{0, A, B, E, F, W, X, Y, Z, S\}$ ,  $T = \{0\}$ , and P (note shrinking productions)

$S \rightarrow FA$      $S \rightarrow FBA$      $FB \rightarrow FOEBO$      $EB \rightarrow XBY$      $OX \rightarrow OX$   
 $EB \rightarrow 0$      $YO \rightarrow OY$      $FX \rightarrow FOW$      $YA \rightarrow ZOA$      $WO \rightarrow OW$   
 $OZ \rightarrow ZO$      $WBZ \rightarrow EB$      $F \rightarrow 1$      $A \rightarrow 1$

Derivation:  $S \Rightarrow FA \Rightarrow^* \lambda\lambda$  gives  $\lambda$ .

S FBA FOEBOA FOXBYOA \* FXOBOYA \* FOWOBOZOA  
\* FOOWBZOOA FOOEB00A F00000A \* 00000

(Note X,Y,W,Z march across the 0 string, adding one on each side. Very TM- like.

# TURING MACHINE

Finite alphabet, Finite number of non-blank cells, finite but unbounded memory. Cell blank or one symbol. FSA reads one cell at any given moment, then either halts or takes these actions:

1. Print a symbol from alphabet on the cell read (possibly same symbol).
2. Go to next state (could be same)
3. Move the Read-Write head one cell left or right.

Describe by set of quintuples  $\{s, r, p, n, d\}$  for (current state, current symbol, symbol printed, next state, direction). In a deterministic TM no two quintuples have same  $s, r$ . In nondeterministic, there can be a choice of actions: this models parallelism or precognition but doesn't add power (new languages accepted).