

Here's how we can do it (example):



The "denotes" function, say I , is called an interpretation.

We say $\text{Dog}(\text{Snoopy})$ is true under interpretation I iff

$$I(\text{Snoopy}) \in I(\text{Dog})$$

Clearly this is the case for $\text{Dog}(\text{Snoopy})$.

We write (as a first approximation)

$$\models_I \text{Dog}(\text{Snoopy}).$$

For the given I , it's also clear that

$$\models_I \text{Has-Tail}(\text{Snoopy})$$

It's also the case (less obviously...) that

$$\models_I (\forall x (\text{Dog}(x) \Rightarrow \text{Has-Tail}(x)))$$

Truth conditions for \forall -- informally:

$I(\text{Dog}) \subseteq I(\text{Has-Tail})$, i.e., the set of dogs is a subset of the things with property Has-Tail.

So, $I(\text{Snoopy}) \in I(\text{Dog}) \subseteq I(\text{Has-Tail})$

Another interpretation, I' :

Snoopy $\xrightarrow{I'}$ 4

Dog $\xrightarrow{I'}$ $\{4, 8, 12, 16, \dots\}$

Has-Tail $\xrightarrow{I'}$ $\{1, 4, 6, 8, 9, 10, 12, 14, 15, \dots\}$

All 3 formulas are again true under I' !

Formal "soundness" (truth-preservation) claim about the original inference:

For any interpretation I of Snoopy, Dog, Has-Tail (where I interprets Snoopy as an individual and Dog, Has-Tail as sets of individuals), if

$\models_I \text{Dog}(\text{Snoopy})$ and $\models_I (\forall x \text{Dog}(x) \Rightarrow \text{Has-Tail}(x))$

then

$\models_I \text{Has-Tail}(\text{Snoopy})$

Let's get a little more precise & general.

First, before we talk about interpretations of particular symbols we should fix the domain \mathcal{D} of all the things we may wish to talk about in our FO language.

Then we interpret individual constants τ as individuals in \mathcal{D} , i.e.,

$I(\tau) \in \mathcal{D}$ also written $\tau^I \in \mathcal{D}$

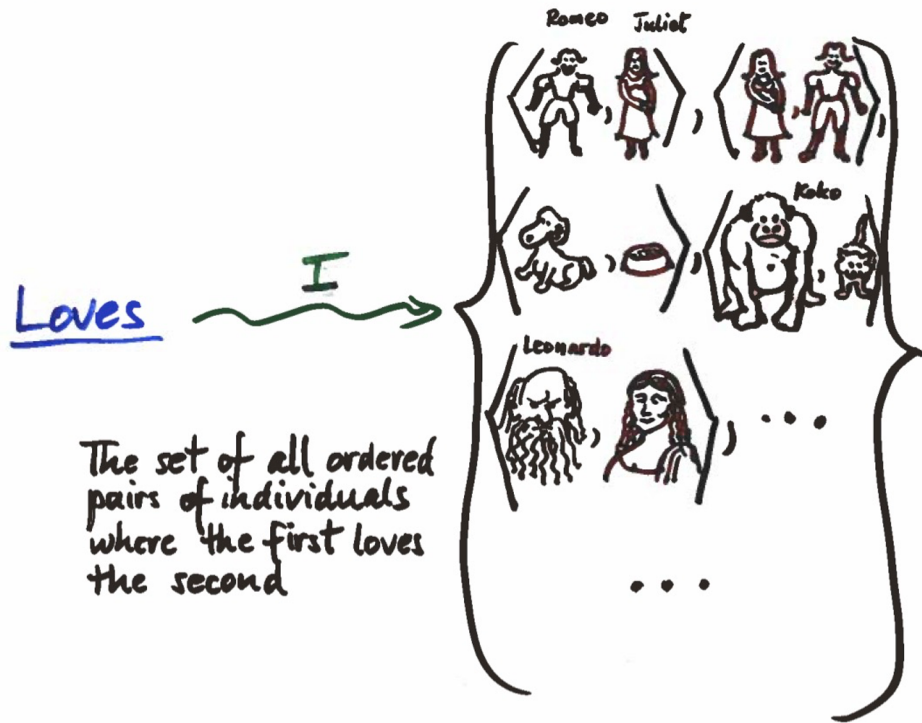
We interpret monadic (1-place) predicate constants π as sets of individuals in \mathcal{D} , i.e.,

$I(\pi) \subseteq \mathcal{D}$ also written $\pi^I \subseteq \mathcal{D}$

For truth, we will now write

$\models_M \varphi$,
where $M = (\mathcal{D}, I)$, the model.

What is $I(\pi)$ for 2-place, 3-place, etc. predicates?



Given this interpretation of 'Loves', and given

$\text{Boy}(\text{Paris}), \forall x. \text{Boy}(x) \Rightarrow \text{Loves}(x, \text{Juliet}),$

Can we semantically justify the conclusion

$\text{Loves}(\text{Paris}, \text{Juliet}),$ i.e.,

$\langle I(\text{Paris}), I(\text{Juliet}) \rangle \in I(\text{Loves})?$

The first formula says $I(\text{Paris}) \in I(\text{Boy});$

The second formula says that the set $I(\text{Boy})$

is a subset of $\{b \mid \langle b, j \rangle \in I(\text{Loves})\}$

(where $j = \text{Juliet} = I(\text{Juliet}).$)

So,

$I(\text{Paris}) \in I(\text{Boy}) \subseteq \{b \mid \langle b, j \rangle \in I(\text{Loves})\}$

To specify truth conditions for ' \forall ' formally requires an inductive truth definition - covered in CSC 244.

$$I(\text{Loves}) \subseteq \mathcal{D} \times \mathcal{D} = \mathcal{D}^2$$

\uparrow
or Loves^I

$$= \{\langle d_1, d_2 \rangle \mid d_1, d_2 \in \mathcal{D}\}$$

$\models_{\mathcal{M}} \text{Loves}(\text{Romeo}, \text{Juliet})$ iff
 $\langle \text{Romeo}^I, \text{Juliet}^I \rangle \in \text{Loves}^I$

First-order logic vs. a first-order language
 fixed vocabulary of constants

