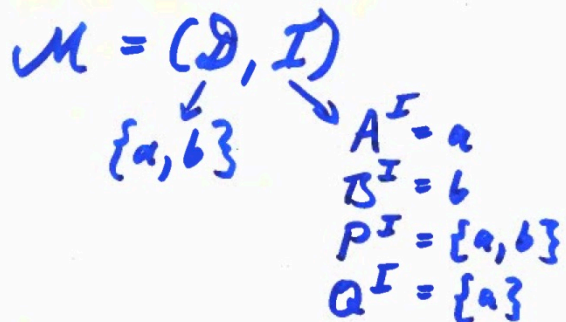


VALIDITY & ENTAILMENT

A model (example)



for FO language with:
 ind. const.: A, B
 pred. const.: P, Q (1-place)
 func. const.: none

Truth in a model (examples)

$\models_{\mathcal{M}} \varphi$ " φ is true in \mathcal{M} " } satisfaction
 " \mathcal{M} is a model of φ " } of φ by all
 v.a. U , rel. to \mathcal{M}

$\models_{\mathcal{M}} (\forall x P(x))$ for \mathcal{M} as above
 (proved on board & in handout)

$\models_{\mathcal{M}} (\exists x Q(x))$ — " —

$\models_{\mathcal{M}} (\forall x (Q(x) \Rightarrow P(x)))$ — " —

Generalize:
 $\models_{\mathcal{M}} \Delta$ iff
 for all $\varphi \in \Delta$,
 $\models_{\mathcal{M}} \varphi$

Note distinction in truth conditions between

"All P are Q" "Everything is P"
 "Some P are Q" "Something is P"

Validity $\models \varphi$

= truth in all models
 = satisfaction by all v.a.'s rel. to all models
 also called logical truth (independently of "how the world is")

$\models \varphi$ iff for all models \mathcal{M} , $\models_{\mathcal{M}} \varphi$
 iff for all v.a. U & all models \mathcal{M} ,
 $\models_{\mathcal{M}} \varphi[U]$

Examples

schemas $\varphi \vee \neg \varphi$ $(\forall x (x = x))$ $P(c) \Leftrightarrow \neg \neg P(c)$
 $\varphi \vee \neg \varphi$ $(\forall v (v = v))$ $\varphi \Leftrightarrow \neg \neg \varphi$

prove $\models (\varphi \vee \neg \varphi)$

Unsatisfiability (logical falsehood)

φ unsatisfiable
 iff for all models \mathcal{M} , $\not\models_{\mathcal{M}} \varphi$
 iff for all models \mathcal{M} , $\models_{\mathcal{M}} \neg \varphi$
 i.e., false in all models,
 i.e., negation true in all models } $\models \neg \varphi$ ^{Assume $(\varphi$ closed)}

$\varphi \wedge \neg \varphi$, $\neg(C \cdot C)$, $(\varphi \Rightarrow \psi) \wedge \varphi \wedge \neg \psi$, etc

Generalize: Δ unsatisfiable iff
 for all models \mathcal{M} , $\not\models_{\mathcal{M}} \Delta$ (i.e.,
 for some $\varphi \in \Delta$, $\not\models_{\mathcal{M}} \varphi$)

Contingent formulas

dependent on "how things are"

φ contingent iff for some model \mathcal{M} ,
 $\models_{\mathcal{M}} \varphi$, and for some model \mathcal{M}' , $\not\models_{\mathcal{M}'} \varphi$
 $P(A)$, Loves(A, B), $\forall x. P(x) \Rightarrow Q(x)$, ...
 ↑
 prove contingent

Satisfiability

φ satisfiable iff for some model \mathcal{M} ,
 $\models_{\mathcal{M}} \varphi$
 i.e., φ is either valid or contingent

Entailment $\Delta \models \varphi$
(logical consequence)

$\Delta \models \varphi$ iff all models of Δ are models of φ
 i.e., iff for all models \mathcal{M} s.t. $\models_{\mathcal{M}} \Delta$,
 we also have $\models_{\mathcal{M}} \varphi$
 i.e., iff for all models \mathcal{M} s.t. for all v.a. \mathcal{U} ,
 $\models_{\mathcal{M}} \Delta[\mathcal{U}]$, we also have that for all
 v.a. \mathcal{U} , $\models_{\mathcal{M}} \varphi[\mathcal{U}]$.

$\neg \neg \varphi \models \varphi$, $\{\varphi, \varphi \Rightarrow \psi\} \models \psi$,

$\forall x P(x) \models P(A)$, $\underbrace{P(A) \wedge \neg P(A)}_{\text{true in no models}} \models Q(B)$